# Hypergraph 2-coloring

## The best known algorithm and its limitations

Grzegorz Ryn

April 11, 2025

# Graphs and hypergraphs

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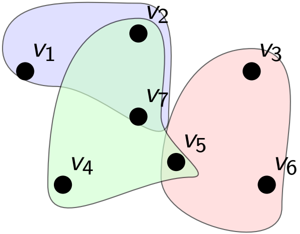

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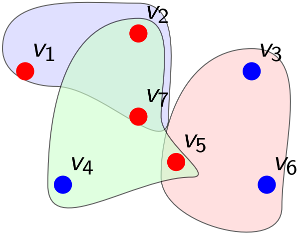

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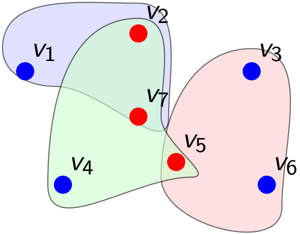

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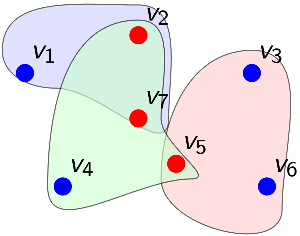

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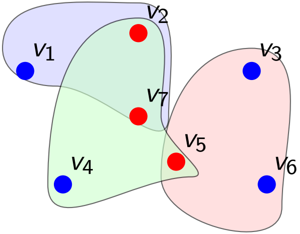


A hypergraph with all edges of size $k$ is called $k$-uniform or $k$-graph.

# Random hypergraphs

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We can generalize it further to non-uniform random hypergraphs by considering each size of (allowed) edge independently.

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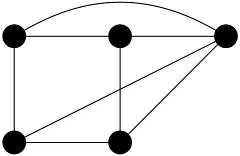

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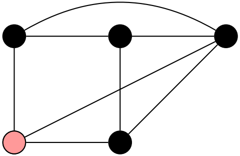

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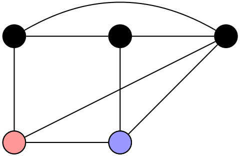

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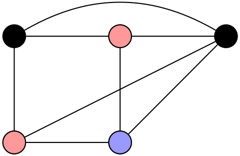

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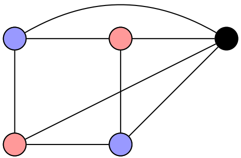

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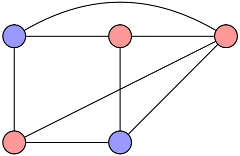

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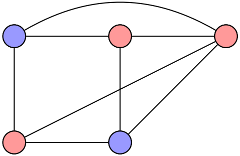

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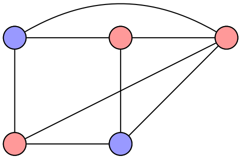

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## $k$-uniform graph 2-colorability

In 1973, L. Lovász showed, that for $k \geq 3$, 2-colorability of $k$-uniform Hypergraph is NP-complete.

# Setting

We are interested in an "assymptotic" setting. Let $k$ be fixed and $n$ arbitrarily large. Obviously hypergraphs drawn from $H_k(n, 0)$ are 2-colorable with high probability but drawn from $H_k(n, 1)$ aren't.

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What is the maximal value of $p = p(n)$ s.t. hypergraph drawn from $H_k(n, p)$ is (algorithmically) 2-colorable with high probability?

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As we might find out, working explicitly with the value of $p$ isn't convenient. Instead we will parametrize our random graph model by the expected number of edges.

# Boundaries

First, we may recall the upper bound. It is based on an idea of Erdős, but in this setting was shown by Alon and Spencer that:

## Upper bound, Erdős 1964

Random hypergraph on $n$ vertices with $\frac{\ln 2}{2} \cdot 2^k \cdot n$ edges on average is with high probability not-2-colorable.

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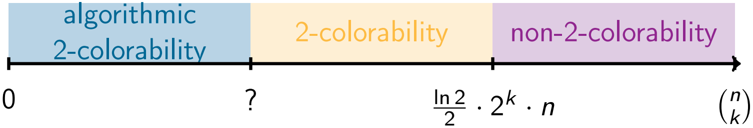
The search for lower bound has been more convoluted. There have been a number of improvments, with the last big improvments in 2001 by Achlioptas and Moore.

**Lower bound, Achlioptas and Moore 2001**

Random hypergraph on $n$ vertices with $\left(\frac{\ln 2}{2} \cdot \left(2^k - 1\right) - \varepsilon\right) \cdot n$ edges on average is 2-colorable with high probability.

# Bird's-eye view

Knowing the threshold at which random hypergraphs stop being 2-colorable, we may ask 'How dense graph can we color with *fast* algorithm?



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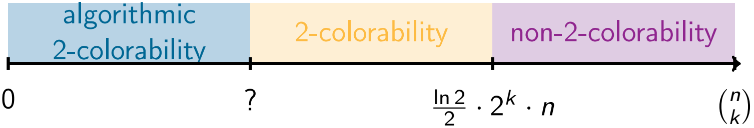


On the sidenote: on the graphic above the intervals corresponding to *2-colorability* and *non-2-colorability* are drawn just after each other. It is not only a simplification, but a reflection of how rapidly our model changes its behaviour. In fact if we look at the values of $p$ for which random graphs are neither 2-colorable or non-2-colorable with high probability, the length of such interval is completely negligible.

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Having build up some context, we may introduce the best algorithm for hypergraph 2-coloring. It was introduced in 1999 by Achlioptas, Kim, Krivelevich and Tetali, hence we will refer to it as **AKKT**.

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The AKKT algorithm suceeds (produces a proper 2-coloring) for random hypergraphs on $n$ vertices with at most $\lambda \cdot \frac{2^k}{k} \cdot n$ edges on a average.

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Moreover, at the time of its first appearance, this threshold was not only the best-known threshold for algorithmic 2-colroability, but for 2-colorability in general.

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We can think of the currently existing 2 and 3 tails as an auxilary hypergraph (on uncolored vertices) with edges of size 2 and 3.

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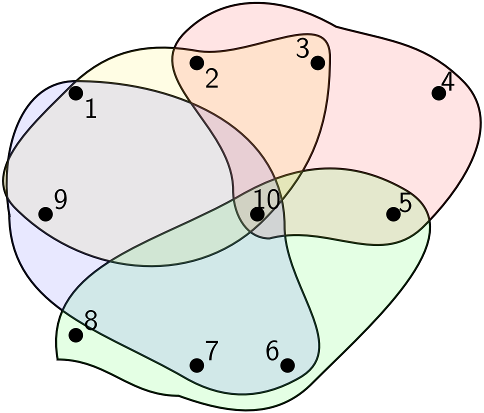

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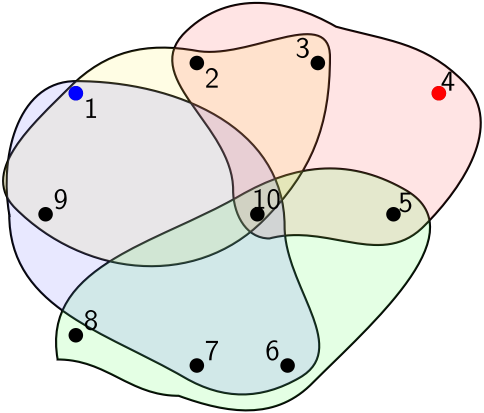

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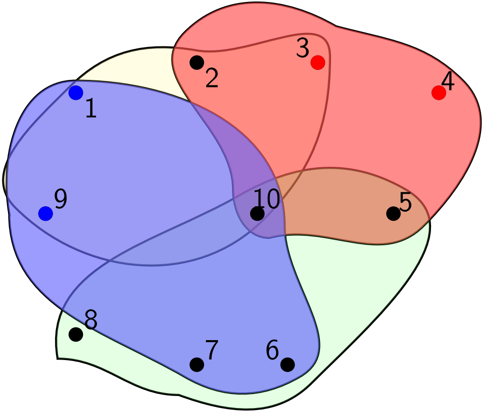


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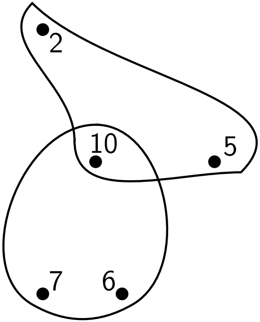

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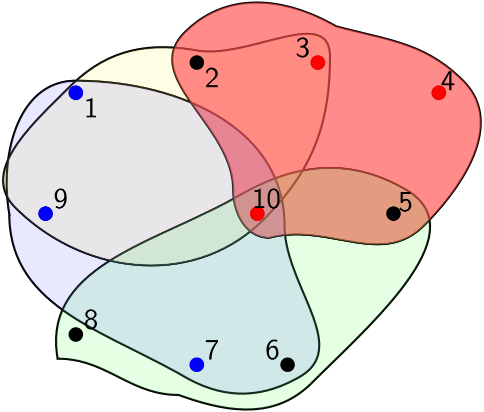


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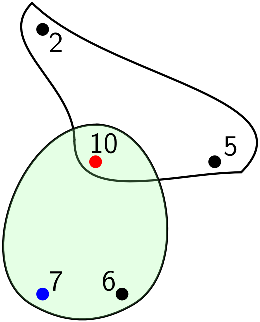

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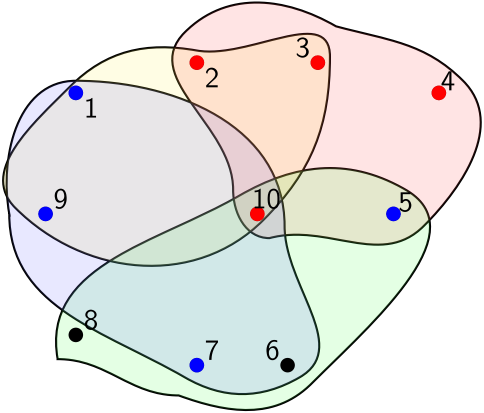


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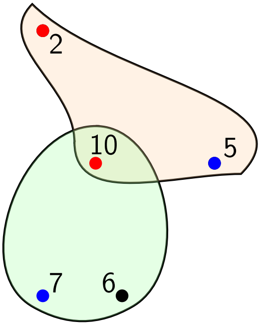

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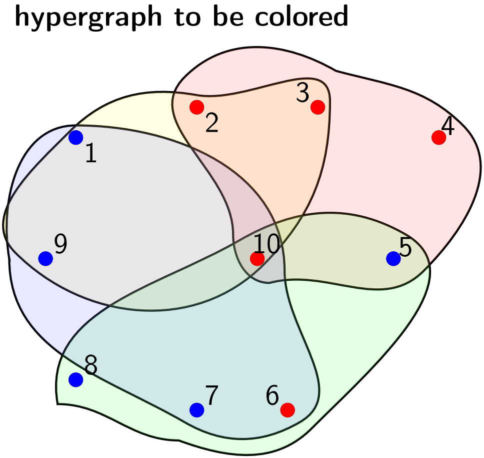


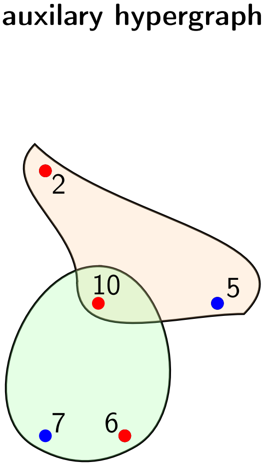

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Let's consider a hypergraph on uncolored vertices only. In each of $n/2$ round of the algorithm, two vertices are collored (differently) and a random number of new 3-edges appear in a narrowed graph.

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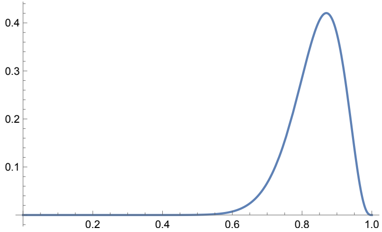


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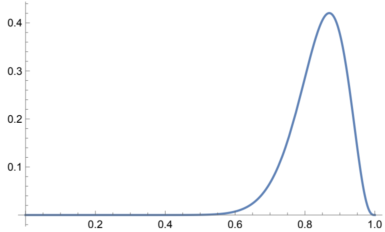


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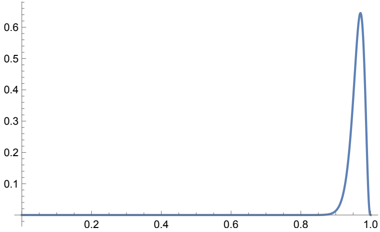


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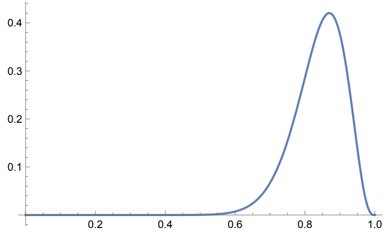


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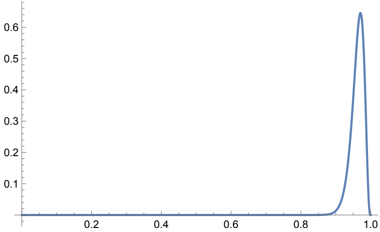


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It can be shown, that we can w.h.p. 2-color our hypergraph if at any point, the expected number of new edges is less than 1.

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Let's work with the non-uniform model allowing for edges of sizes $k_1, k_2, \ldots, k_l$ with expectancy $\lambda_1 \cdot \frac{2^{k_1}}{k_1} \cdot n, \lambda_2 \cdot \frac{2^{k_2}}{k_2} \cdot n, \ldots$.

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**AKKT, non-uniform case**

For any $c > 0$, there exists a corresponding random hypergraph model (a sequence of $(k_i, \lambda_i)$), s.t: $\sum \lambda_i \geq c$

# non-uniform case - upper bound

Let's recall the upper bound (non-constructive) for 2-colorability:

**Upper bound, Erdős 1964**

Random hypergraph on $n$ vertices with $\frac{\ln 2}{2} \cdot 2^k \cdot n$ edges on average is with high probability not-2-colorable.

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**non-uniform upper bound**

Random non-uniform hypergraph drawn from a model described above is with high probability non-2-colorable if $\sum \lambda_i \geq \frac{\ln 2}{2}$.

# interesting contrast

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We can think of $2^k \cdot n$ and $\frac{2^k}{k} \cdot n$ as of an assymptotic order of the number of edges *critical* in critical hypergraphs - for 2-colorability and algorithmic 2-colorability respectively.

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## Algorithmic vs. non-constructive upper bound

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It shows, that while 2-colorability threshold translates 1:1 between uniform and non-uniform case. Moreover hypergraphs are in some sense invariant to the type of an edge, while there's some space left in algorithmic coloring.

# Algorithmic barrier

To conclude, we may ask the question - can we color denser hypergraphs?

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We can consider the space of all $2^n$ 2-colorings of $n$-vertex hypergraph. In 2008, Coja-Oghlan published a paper in which he analyses how the geometry of this space changes with increasing $p$.

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It happens, that for $p$ up to around $\frac{2^k}{k} \cdot n$ the space of proper colorings is connected in the sense of Hamming distance. Beyond this threshold, this space becomes *shattered*. From the connected space, it breaks into exponentionally many small pieces, which are very far apart from each other - linearly in terms of vertices.